Renewable and Sustainable Energy Reviews 16 (2012) 5363–5378

Contents lists available at SciVerse ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser

A review on natural convective heat transfer of nanofluids

Zoubida Haddad [a,b], Hakan F. Oztop [b], Eiyad Abu-Nada [c,*], Amina Mataoui [a]

[a] *Department of Fluid Mechanics, Faculty of Physics, University of Sciences and Technology-Houari Boumediene, Algiers, Algeria*
[b] *Department of Mechanical Engineering, Technology Faculty, Firat University, Elazig, Turkey*
[c] *Department of Mechanical Engineering, King Faisal University, Al-Ahsa 31982, Saudi Arabia*

ARTICLE INFO

Article history:
Received 6 September 2011
Accepted 1 April 2012
Available online 6 July 2012

Keywords:
Nanofluids
Natural convection
Heat transfer

ABSTRACT

Nanofluids are considered to have great potential for heat transfer enhancement and are highly suited to application in practical heat transfer processes. Recently, several important studies were carried out to understand and explain the causes of the enhancement or control of heat transfer using nanofluids. The main aim upon which the present work is based is to give a comprehensive review on the research progress on the natural convective heat transfer characteristics of nanofluids for both single- and two-phase models. Both experimental and theoretical studies are reviewed for natural convection of nanofluids in different types of enclosures.

Contents

1. Introduction 5363
2. Brief theory 5364
 2.1. Model description 5364
 2.2. Conservation equations 5364
  2.2.1. Single phase model 5364
  2.2.2. Two-phase model 5364
 2.3. Physical properties of the nanofluids for single-phase model 5365
  2.3.1. Density 5365
  2.3.2. Specific heat capacity 5365
  2.3.3. Dynamic viscosity and thermal conductivity 5365
3. Numerical investigations 5365
 3.1. Single phase model 5365
 3.2. Two phase model 5370
4. Experimental investigation 5370
5. Recent Studies 5370
6. Conclusion 5376
References 5376

1. Introduction

Natural convection heat transfer is an important phenomenon in engineering and industry with widespread application in diverse field, such as, geophysics, solar energy, electronic cooling, nuclear energy and other industrial system in various sectors. A major limitation against enhancing the heat transfer in such engineering systems is the inherently poor thermal conductivity of conventional fluids, including oil, water and ethylene glycol mixture. Therefore, for more than a century since Maxwell's theories in 1873, scientists and engineers have made great effort to break this fundamental limit by dispersing millimeter-or micrometer sized particles in liquids. However, the major problem with the use of such large particles is the rapid sedimentation of these particles in fluids.

Maxwell's concept is old but what is new and innovative in the concept of nanofluids is the idea of using nanometer-sized

* Corresponding author.
E-mail address: eabunada@kfu.edu.sa (E. Abu-Nada).

1364-0321/$ - see front matter
http://dx.doi.org/10.1016/j.rser.2012.04.003

Nomenclature

c_p	specific heat at constant pressure (J kg^{-1} K^{-1})
D_T	thermal diffusion coefficient (m^2/s)
d	diameter (m)
g	gravitational acceleration (m s^{-2})
H	dimensional cavity height (m)
k	thermal conductivity (W/m K)
k_B	Boltzmann constant (J/K)
p	pressure (Pa)
Pr	Prandtl number, $\mathrm{Pr} = \nu_f/\alpha_f$
Pe	Peclet number
Re	Reynolds number, $\mathrm{Re} = \rho_f k_B T/3\pi\mu_f^2 l_f$
T	dimensional temperature (°C)
v	dimensional nanofluid velocity (m s^{-1})

Greek symbols

μ	dynamic viscosity (Ns m^{-2})
τ	stress tensor (Pa)
α	thermal diffusivity (m^2 s^{-1})
ϕ	dimensional nanoparticle volumetric fraction
ϕ	inclination angle
ρ	density (kg m^{-3})
η	ratio of the nanolayer thickness to the original particle radius

Subscripts

f	base fluid
nf	nanofluid
p	particle
k	thermal conductivity (W/m K)

particles to create stable and highly conductive suspensions, primarily for suspension stability and for dynamic thermal interactions. Recognizing an excellent opportunity to apply nanotechnology to thermal engineering, Eastman et al. [1] is the first who conceived the novel concept of nanofluid by hypothesizing that it is possible to break down these century-old technical barriers by exploiting the unique properties of nanoparticles.

Nanofluids are dilute suspensions of nanoparticles smaller than 100 nm. The thermal conductivity of the particle materials, metallic or nonmetallic such as Al_2O_3, CuO, Cu, SiO and TiO_2 are typically order-of-magnitude higher than the conventional fluid. Such nanofluids represent a class of its own different from those of traditional solid-mixtures, they have superior properties like high thermal conductivity [1–5], minimal clogging in flow passage, reduced puming power, long term stability and homogeneity. Therefore nanofluids have a wide range of potential application where improved heat transfer is required.

The purpose of this paper first is to review the available results of the flow and heat transfer presented in literature for natural convection of nanofluids, including the experimental and theoretical investigations for both single- and two-phase models. The results of massive number of different parameters affect the fluid flow and heat transfer characteristics are summarized. The second purpose is to explore the causes of the enhancement of heat transfer, which would be of help in understanding the transport in nanofluids, and accurately predict natural convection using nanofluids.

2. Brief theory

2.1. Model description

In the literature, convective heat transfer with nanofluids can be modeled using mainly the two-phase or single approach. In the two-phase approach, the velocity between the fluid and particles might not be zero [6] due to several factors such as gravity, friction between the fluid and solid particles, Brownian forces, Brownian diffusion, sedimentation and dispersion. In the second approach, the nanoparticles can be easily fluidized and therefore, one may assume that the motion slip between the phases, if any would be considered negligible [7]. The latter approach is simpler and more computationally efficient.

2.2. Conservation equations

2.2.1. Single phase model

Although nanofluids are solid–liquid mixtures, the approach conventionally used in most studies of natural convection handles the nanofluid as a single-phase (homogenous) fluid. In fact, due to the extreme size and low concentration of the suspended nanoparticles, the particles are assumed to move with same velocity as the fluid. Also, by considering the local thermal equilibrium, the solid particle–liquid mixture may then be approximately considered to behave as a conventional single-phase fluid with properties that are to be evaluated as functions of those of the constituents.

The governing equations for a homogenous analysis of natural convection are continuity, momentum, and energy equations with their density, specific heat, thermal conductivity, and viscosity modified for nanofluid application. The specific governing equations for various studied enclosures are not shown here and they can be found in different references [18,32,33,55].

2.2.2. Two-phase model

Several authors have tried to establish convective transport models for nanofluids. Nanofluid is a two-phase mixture in which the solid phase consists of nano-sized particles. In view of the nanoscale size of the particles, it may be questionable whether the theory of conventional two-phase flow can be applied in describing the flow characteristics of nanofluid. On the other hand, several factors such as gravity, friction between the fluid and solid particles and Brownian forces, the phenomena of Brownian diffusion, sedimentation, and dispersion may affect a nanofluid flow. Consequently, the slip velocity between the fluid and particles cannot be neglected for simulating nanofluid flows. Since the two phase approach considers the movement between the solid and fluid molecule, it may have better prediction in nanofluid study. To fully describe and predict the flow and behavior of complex flows, different multiphase theories have been proposed and used. The large number of published articles concerning multiphase flows typically employed the Mixture Theory to predict the behavior of nanofluids [8–10].

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [11], using a model in which Brownian motion and thermophoresis are accounted for. Buongiorno

developed a two-component four-equation non-homogeneous equilibrium model for mass, momentum, and heat transfer in nanofluids. The nanofluid is treated as a two-component mixture (base fluid + nanoparticles) with the following assumptions:

- No chemical reactions.
- Negligible external forces.
- Dilute mixture ($\phi = 1$).
- Negligible viscous dissipation.
- Negligible radiative heat transfer.
- Nanoparticle and base fluid locally in thermal equilibrium.

Invoking the above assumptions, the following equations represent the mathematical formulation of the non- homogenous single phase model for the governing equations as formulated by Buongiorno [11]:

2.2.2.1. Continuity equation.

$$\nabla \cdot v = 0 \tag{1}$$

2.2.2.2. Nanoparticle continuity equation.

$$\frac{\partial \phi}{\partial t} v \cdot \nabla \phi = \nabla \cdot \left(D_B \nabla \phi + D_T \frac{\nabla T}{T} \right) \tag{2}$$

Here ϕ is nanoparticle volume fraction, D_B is the Brownian diffusion coefficient given by the Einstein–Stokes's equation:

$$D_B = \frac{k_B T}{3\pi \mu d_p} \tag{3}$$

where μ is the viscosity of the fluid and d_p is the nanoparticle diameter and D_T is the thermophoretic diffusion coefficient, which is defined as

$$D_T = \left(\frac{\mu}{\rho} \right) \left(0.26 \frac{k}{2k + k_p} \right) \phi \tag{4}$$

In Eq. (5), k and k_p are the thermal conductivity of the fluid and particle materials, respectively.

2.2.2.3. Momentum equation.

$$v \cdot \nabla v = -\frac{1}{\rho_{nf}} \nabla p + \nabla \cdot \tau + g \tag{5}$$

where

$$\tau = -\mu_{nf} (\nabla v + (\nabla v)^t) \tag{6}$$

where the superscript 't' indicates the transpose of ∇v.

2.2.2.4. Energy equation.

$$v \cdot \nabla T = \nabla (\alpha_{nf} \nabla T) + \frac{\rho_p c_p}{\rho_{nf} c_{nf}} \left(D_B \nabla \phi \cdot \nabla T + D_T \frac{\nabla T \cdot \nabla T}{T} \right) \tag{7}$$

This nanofluid model can be characterized as a 'two-fluid' (nanoparticles + base fluid), four-equation (2 mass + 1 momentum + 1 energy), non-homogeneous (nanoparticle/fluid slip velocity allowed) equilibrium (nanoparticle/fluid temperature differences not allowed) model.

Note that the conservation equations are strongly coupled. That is, v depends on ϕ via viscosity; ϕ depends on T mostly because of thermophoresis; T depends on ϕ via thermal conductivity and also via the Brownian and thermophoretic terms in the energy equation: ϕ and T obviously depends on v because of the convection terms in the nanoparticle continuity and energy equations, respectively.

2.3. Physical properties of the nanofluids for single-phase model

Base nanofluid properties have been published over the past few years in literature. However, only recently have some data on temperature-dependent properties been provided, even though they are only for nanofluid effective thermal conductivity and effective absolute viscosity.

2.3.1. Density

In the absence of experimental data for nanofluid densities, constant-value temperature independent values, based on nanoparticle volume fraction, are used:

$$\rho_{nf} = (1-\phi)\rho_f + \phi \rho_p$$

2.3.2. Specific heat capacity

It has been suggested that the effective specific heat can be calculated using the following equation as reported in Refs. [12–15] as

$$(C_p)_{nf} = (1-\phi)(C_p)_f + \phi (C_p)_p$$

Other authors suggest an alternative approach based on heat capacity concept [7,16]:

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi (\rho C_p)_p$$

These two formulations may of course lead to different results for specific heat. Due to the lack of experimental data, both formulations are considered equivalent in estimating nanofluid specific heat capacity [14].

2.3.3. Dynamic viscosity and thermal conductivity

In this work, different nanofluid models based on a combination of the different formulas for the dynamic viscosity and thermal conductivity adopted in the studies of natural convection are summarized in Tables 1 and 2.

3. Numerical investigations

3.1. Single phase model

Khanafer et al. [17] were the first who studied numerically, the buoyancy driven heat transfer enhancement of nanofluid in a square cavity. They used Wasp's model [18] for thermal conductivity and Brinkman model [19] for viscosity. They illustrated that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. In addition, they indicated that the nanofluid heat transfer rate increases with an increase in the nanoparticles volume fraction. They also showed that the presence of nanoparticles in the fluid is found to alter the structure of the fluid flow.

Jang and Choi [20] examined the thermal characteristics of free convection in rectangular cavity heated from below with nanofluid such as water based nanofluids containing 6 nm copper and 2 nm diamond, using Jang and Choi's model [21] for the effective thermal conductivity. They concluded that the nanofluids is more stable and have larger heat transfer coefficient compared with a classical fluid in a rectangular cavity with free convection.

Kim et al. [22] analyzed the convective instability driven by buoyancy and heat transfer characteristics of nanofluids. They chose Wang et al. [23] model based on the mean field approach for expressing the thermal conductivity enhancement. They indicated that as the density and heat capacity of nanoparticles increase and the thermal conductivity and the shape factor of nanoparticles decrease, the convective motion in a nanofluid sets

Table 1
Various viscosity models for nanofluids.

Models	Dynamic viscosity
Brinkman model [19]	$\mu_{nf} = \dfrac{\mu_f}{(1-\phi)^{2.5}}$
Einstein model [27]	$\mu_{nf} = \mu_f(1+2.5\phi) \quad \phi \prec 0.05$
Pak and Cho's Correlation [28]	$\mu_{nf} = \mu_f(1+39.11\phi+533.9\phi^2)$
Nguyen et al. model [36] (fitted by Abu-Nada [34])	$\mu_{Cuo} = -0.6967+\dfrac{15.937}{T}+1.238\phi+\dfrac{1356.14}{T^2} -0.259\phi^2-30.88\dfrac{\phi}{T}-\dfrac{19652.74}{T^3}+0.01593\phi^3+4.38206\dfrac{\phi^2}{T}+147.573\dfrac{\phi}{T^2}$ $\mu_{Al_2O_3} = \exp(3.003-0.04203T-0.5445\phi+0.0002553T^2+0.0524\phi^2-1.622\phi^{-1})$
Jang et al. model [41]	$\mu_{nf} = \mu_f(1+2.5\phi)\left[1+\eta\left(\dfrac{d_p}{H}\right)^{-2\varepsilon}\phi^{2/3}(\varepsilon+1)\right]$
Koo and Kleinstreuer [48]	$\mu_B = 5\times10^4\beta\phi\rho_f\sqrt{\dfrac{k_BT}{d_p\rho_p}}f(T,\phi)$ $\beta = 0.0137(100\phi)^{-0.8229} \quad for \quad \phi \prec 1\%$ $\beta = 0.0011(100\phi)^{-0.7272} \quad for \quad \phi \succ 1\%$ $f(T,\phi) = (-6.04\phi+0.4705)T+(1722.3\phi-134.63)$ $1\% \le \phi \le 4\% \quad and \quad 300\ \mathrm{k} \le T \le 325\ \mathrm{k}$
Maiga model [81]	$\mu_{nf} = \mu_f(1+7.3\phi+123\phi^2)$
Brownian model [82]	$\mu_{nf} = \mu_f(1+2.5\phi+6.17\phi^2)$
Nguyen model [83]	$\mu_{nf} = \mu_f(1+0.025\phi+0.015\phi^2)$
Gherasim et al. [84]	$\dfrac{\mu_{nf}}{\mu_f} = 0.904e^{14.8\phi}$

in easily. They also showed that heat transfer coefficient of a nanofluid is enhanced by all parameters with respect to the volume fraction.

Jou and Tzeng [24] numerically studied the natural convection heat transfer enhancements of nanofluids within a two-dimensional enclosure. They analyzed heat transfer performance using Khanafer's model for various parameters, like volume fraction, Grashof number, and aspect ratio of the enclosure. Results showed that increasing the buoyancy parameter and volume fraction of nanofluids cause an increase in the average heat transfer coefficient.

Polidori et al. [25] analytically investigated the natural convection heat transfer of Newtonian nanofluids over a vertical semi-infinite plate. In their study, the dealt with γ-Al_2O_3/water nanofluids whose Newtonian behavior was experimentally confirmed for particle volume fractions less than 4%. It was shown that special care has to be exercised in drawing generalized conclusions about the heat transfer enhancement. They found that natural convection heat transfer is not solely characterized by the nanofluid effective thermal conductivity and that the sensitivity to the viscosity model used seems undeniable and play a key role in the heat transfer behavior.

Hwang et al. [26] theorically investigated the thermal characteristics of natural convection in a rectangular cavity heated from below with water-based nanofluids containing alumina (Al_2O_3). They used thermal conductivity model of Jang and Choi [21] and various models for the effective viscosity. It was shown that the experimental results are put between two theoretical lines derived from Jang and Choi's conductivity model used along with both Einstein's model [27] and Pak and Cho's correlation [28] for viscosity. They showed that water-based Al_2O_3 nanofluids is more stable than base fluid in a rectangular cavity heated from below as the volume fraction of nanoparticles increases, the size of nanoparticles decreases, or the average temperature of nanofluid increases. They also showed that the ratio of heat transfer coefficient of nanofluids to that of base fluid is decreased as the size of nanoparticles increases, or the average temperature of nanofluids is decreased. In addition, they stated that the effective thermal conductivity models used by Kim et al. [22] and Khanafer [17] cannot predict the effect of the size of nanoparticles and the average temperature of nanofluids on the enhancement of the effective thermal conductivity, although the models are able to predict the effect of the particle fraction of nanoparticles on the enhancement of the effective thermal conductivity of nanofluids.

Ho et al. [29] aimed to clarify and present a deep explanation on the influences of uncertainties due to adopting various formulas for the effective thermal conductivity and dynamic viscosity. They presented a numerical analysis for natural convection of water Al_2O_3 nanofluid in a square vertical enclosure. They observed that a significant difference between enhancements in the dynamic viscosity estimated from the two adopted formulas leads to contradictory heat transfer efficacy of the nanofluid, so that the heat transfer across the enclosure can be found to be enhanced or migtated with respect to the base fluid. According to the authors, the contradictory behavior dictated by the significant difference between the dynamic viscosity enhancements calculated, using two formulas might contribute to explaining the disparate findings among the numerical predictions [17] and experimental results [76,77].

Oztop and Abu-Nada [30] investigated heat transfer and fluid flow due to buoyancy forces in a partially heated enclosure using different

Table 2
Various thermal conductivity models for nanofluids.

Model	Thermal conductivity
Wasp model [18]	$\frac{k_{nf}}{k_f}=\frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}$
Jang and Choi model [21]	$\frac{k_{nf}}{k_f}=(1-\phi)+Bk_p\phi+18\times10^6\frac{3d_f}{d_{nano}}k_f\mathrm{Re}^2_{d_{nano}}\mathrm{Pr}\phi$
Bruggeman model [23]	$\frac{k_{nf}}{k_f}=\frac{(3\phi-1)k_p/k_f+[3(1-\phi)-1]+\sqrt{\Delta_B}}{4}$ $\Delta_B=[(3\phi-1)k_p/k_f+(3(1-\phi)-1)]^2+8k_p/k_f$
Chon et al. model [35]	$\frac{k_{nf}}{k_f}=1+64.7\phi^{0.7640}\left(\frac{d_f}{d_p}\right)^{0.3690}\left(\frac{k_f}{k_p}\right)^{0.7476}\mathrm{Pr}_T^{0.9955}\mathrm{Re}^{1.2321}$
Koo and Kleinstreuer [48]	$k=k_f\left[\frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}\right]+5\times10^4\beta\phi\rho_f(C_p)_f\sqrt{\frac{k_BT}{d_p\rho_p}}f(T,\phi)$
Charuyakorn et al. [85]	$\frac{k_{nf}}{k_f}=\frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}(1+b\phi Pe_p^m)$
Staionary model [86]	$\frac{k_{nf}}{k_f}=\left[1+\frac{k_p\phi d_f}{k_f(1-\phi)d_p}\right]$
Yu and Choi [87]	$\frac{k_{nf}}{k_f}=\frac{k_p+2k_f-2\phi(k_f-k_p)(1+\eta)^3}{k_p+2k_f+\phi(k_f-k_p)(1+\eta)^3}$
Patel et al. [88]	$\frac{k_{nf}}{k_f}=1+\frac{k_pd_f\phi}{k_fd_p(1-\phi)}\left[1+c\frac{2k_BTd_p}{\pi\alpha_f\mu_fd_p^2}\right]$
Mintsa et al. [89]	$\frac{k_{nf}}{k_f}=1.72\phi+1.0$

nanofluids. The flush mounted heater was located to the left vertical wall with a finite length. The temperature of vertical wall was lower than that of heater while other walls were insulated. Calculations were performed for different Rayleigh numbers, height of heater, location of heater, aspect ratio and volume fraction of nanoparticles. They found that heat transfer increases with increasing Rayleigh number, height of heater and volume fraction of nanoparticles. In addition, the heat transfer enhancement was found to be dependent on the type of nanofluid and more pronounced at low aspect ratio that at high aspect ratio.

Ögüt [31] studied numerically the natural convection heat transfer of water-based nanofluids in an inclined enclosure with a constant flux heater. Five types of nanoparticles were taken into consideration: Cu, CuO, Al_2O_3 and TiO_2. The results showed that the average heat transfer rate increases significantly as Rayleigh number and particle volume fraction increase, the effect is more pronounced for Ag and Cu nanoparticles. The results also showed that the average heat transfer decreases with an increase in the length of the heater. As the heater length is increased, the average heat transfer rate starts to decrease for a smaller inclination angle.

Abu-Nada and Oztop [32] analyzed numerically the effect of inclination angle on natural convection heat transfer and fluid flow in a two dimensional enclosure filled with Cu-nanofluid. It was found that the effect of nanoparticle concentration on Nusselt number is more pronounced at high volume fraction than at low volume fraction. It was also found that the Effects of inclination angle on percentage of heat transfer enhancement become insignificant at low Rayleigh number but it decreases the enhancement of heat transfer with nanofluid.

Abouali and Falahatpisheh [33] carried out a numerical study of internal free convection of Al_2O_3 water nanofluid in vertical annuli. Vertical walls are maintained at constant temperatures and horizontal walls are adiabatic. They developed a correlation as a function of Nusselt number of base fluid and particle fraction which is a monotonically linear decreasing function of particle fraction. In addition, they showed that thermal conductivity model of Wasp [18] greatly overestimates the Nusselt number of experimental data of Putra et al. [76] and the thermal conductivity of Jang and Choi [21] agrees well with the experiment and shows deterioration of natural convection heat transfer.

Abu-Nada [34] studied the heat transfer enhancement in horizontal annuli using variable properties of Al_2O_3–water nanofluid. He used Chon et al. [35] expression for conductivity and Nguyen et al. [36] experimental data for viscosity. They observed that for $\mathrm{Ra}\geq10^4$, the average Nusselt number was reduced by increasing the volume fraction of nanoparticles. However, for $\mathrm{Ra}=10^3$, the average Nusselt number increased by increasing the volume fraction of nanoparticles. For $\mathrm{Ra}=10^4$, the Nusselt number was deteriorated everywhere around the cylinder surface especially at high expansion ratio. However, this reduction is only restricted to certain regions around the cylinder surface at $\mathrm{Ra}=10^3$. They observed also that The Nguyen et al. data and Brinkman model gives completely different predictions for $\mathrm{Ra}=10^4$ where the difference in prediction of Nusselt number reached 30%. However, this difference was less than 10% at $\mathrm{Ra}=10^3$.

Aminossadati and Ghasemi [37] presented a study of natural convection in a partially heated enclosure from below and filled with different types of nanofluids. The effects on the enclosure cooling performance of Rayleigh number, solid volume fraction, heat source length and location and the type of nanofluid were studied. The results indicated that adding nanoparticles into pure water improves its cooling performance especially at low Rayleigh numbers. The type of nanoparticles and the length and

location of the heat source proved to significantly affect the heat source maximum temperature.

Abu-Nada et al. [38] presented a study of the heat transfer enhancement in a differentially heated enclosure using variable thermal conductivity and variable viscosity of Al_2O_3–water and CuO–water nanofluids. The results indicated that for an enclosure with unity aspect ratio, the average Nusselt number of Al_2O_3–water nanofluid at high Rayleigh number was reduced by increasing the volume fraction of nanoparticles above 5%. However, at low Rayleigh number, the average Nusselt number was slightly enhanced by increasing the volume fraction of nanoparticles. Also, at high Rayleigh numbers, Cuo–water nanofluids manifest a continuous decrease in Nusselt number as the volume fraction of nanoparticles is increased. However, the Nusselt number was not sensitive to the volume fraction at low Rayleigh numbers. It was observed that enclosures, having high aspect ratios, experience more deterioration in the average Nusselt number when compared to enclosures having low aspect ratios. It was also observed that the average Nusselt number was more sensitive to the viscosity models than to the thermal conductivity models.

Lin and Violi [39] analyzed the heat transfer and fluid flow of natural convection in a cavity filled with Al_2O_3/water nanofluid that operates under differentially heated walls. The thermal conductivity and dynamic viscosity of the nanofluid are employed by Xu's model [40] and Jang's et al. model [41], respectively. The heat transfer rates were examined for parameter of non-uniform nanoparticle size, mean nanoparticle diameter, nanoparttícle volume fraction, Prandtl number and Grashof number. They illustrated that the heat transfer characteristics of the nanofluid can be enhanced as the mean nanoparticle diameter is decreased from 250 to 5 nm. They attributed these phenomena to the dominant effect of the Brownian motion caused by heat convection. However, the heat transfer performance of the nanofluid was found to be less significant as the dimensionless total thermal conductivity of the nanofluid is close to unity due to the increase of nanoparticle sizes. This contradictory effect of nanofluids was explained to be mainly caused by the effective dynamic viscosity.

Abu-nada and Chamkha [42] focused on the study of the natural convection heat transfer characteristics in a differentially heated enclosure filled with Cuo–Eg–water nanofluid. Overall, the study indicated that the effects of the viscosity models are predicted to be more predominant on the behavior of the average Nusselt number than the influence of the thermal conductivity models. Also, the enclosure aspect ratio is predicted to have significant effects on the behavior of the average Nusselt number which decreases as the enclosure aspect ratio increases.

Abu-nada [43] presented a solution of Rayleigh-Bénard convection problem of CuO–water nanofluids flowing in a rectangular enclosure. The results were presented over a wide range of Rayleigh numbers and volume fractions, using variable thermal conductivity and variable viscosity models. It has been shown that For $Ra > 10^3$, the average Nusselt number was reduced by increasing the volume fraction of nanoparticles. However, for $Ra = 10^3$, the average Nusselt number was enhanced by increasing the volume fraction of nanoparticles. Abu-Nada explained such deterioration in heat transfer observed experimentally and showed that the influence of nanoparticles elucidates two opposing effects on the energy transport: a favorable effect driven by the presence of high thermal conductive nanoparticles and undesirable effect promoted by high level of viscosity experienced at high volume fraction of nanoparticles, where at high Rayleigh number the undesirable effect of high viscosity becomes dominant, which causes deterioration in heat transfer. The results obtained using variable properties of nanofluid were compared with those based on constant property simulations and it was found that the temperature influence is small compared to the influence of high viscosity brought by the presence of high concentration of nanoparticles.

Corcione [44] presented two general correlations for thermal conductivity and viscosity of nanofluids using many available experimental data. These two correlations are sensitive to types of the base fluid and nanoparticle, diameter of nanoparticle and the temperature of the nanofluid. Using these new correlations, Corcione [45] investigated the heat transfer features of buoyancy driven nanofluid inside rectangular enclosures differentially heated at the vertical walls. They assumed that nanofluids behave more like single-phase fluid rather than a conventional solid–liquid mixture, thus, they extended the heat transfer correlations available in literature for single phase natural convection in enclosure to nanoparticle suspension, by replacing the thermophysical properties appearing in them with the nanofluid effective properties calculated at the average temperature. Results showed the existence of an optimal particle loading for maximum heat transfer. Specifically, for any assigned combination of solid and liquid phases, the optimal volume fraction was found to increase slightly with decreasing the nanoparticle size, and to increase much more remarkably with increasing both the nanofluid average temperature and the slenderness of the enclosure.

Ghasemi and Aminossadati [46] presented the results of a numerical study on the natural convection in a right triangular enclosure, with a heat source on its vertical wall and filled with a water–Cuo nanofluid. The viscosity and the thermal conductivity of the nanofluid were determined based on the models presented by Koo and Kleinstreuer [47,48] in which a Brownian motion term was added and valid in the range of $1 \leq \phi \leq 4\%$. It was noticed that when Brownian motion is neglected, the heat transfer rate continuously increases with increasing the solid volume fraction at all Rayleigh numbers, whereas, at high Rayleigh numbers, an optimum solid volume fraction can be found, which results in the maximum heat transfer when Brownian motion is considered.

Santra et al. [49] simulated the behavior of heat transfer due to natural convection in a differentially heated cavity. They calculated the shear stresses using Ostwald–de Waele model [50] for an incompressible non-Newtonian fluid. It was observed that the heat transfer rate decreases with increase in solid volume fraction for a particular Rayleigh number. However, it increases with Rayleigh number for a particular solid volume fraction.

Rashmi et al. [51] performed a numerical study of natural convection for Al_2O_3–water nanofluid inside a horizontal cylinder, using FLUENT commercial code. Stationary particle model was employed for thermal conductivity. The numerical results showed decrease in heat transfer with increase in particle volume fraction. Numerical predictions were also presented to be compared with experimental results of Putra et al. [76]. The comparison showed a good agreement with the experimental findings.

Das and Ohal [52] conducted a numerical investigation of natural convection inside a square cavity containing Cu–water nanofluid with half-active and half-insulated vertical walls. They considered five different cases based on the position of the active walls. It was found that the nanoparticles when immersed in a fluid are capable of increasing the heat transfer capacity of the base fluid and this effect is more pronounced for high solid volume fraction.

Ghasemi et al. [53] investigated the effects of magnetic field on natural convection in a nanofluid-filled square enclosure for different Hartmann number, Rayleigh number and nanoparticle volume fraction. They indicated that the heat transfer rate increases with an increase of the Rayleigh number but it decreases with an increase of the Hartmann number. An increase of the solid volume fraction may result in enhancement or deterioration of the heat transfer performance depending on the value of Hartmann number and Rayleigh numbers.

Oztop et al. [54] studied the problem of steady state natural convection in an enclosure filled with different types of nanoparticles (Al_2O_3 and TiO_2) numerically. It was observed that the addition of nanoparticles into water affects the fluid flow and temperature distribution especially for higher Rayleigh numbers. In addition, an enhancement in heat transfer rate was registered for the whole range of Rayleigh numbers. However, low Rayleigh numbers showed more heat transfer increment compared to high Rayleigh numbers. It was also noticed that the difference between Al_2O_3 and TiO_2 enhancement becomes more pronounced at high volume fractions of nanoparticles.

Aminossadati and Ghasemi [55] performed a numerical study of natural convection in a square cavity filled with a water–CuO nanofluid. They considered two pairs of heat source–sink to cover the entire length of the bottom wall of the cavity while the other walls were thermally insulated. The results showed that regardless of the position of the pairs of source–sink, the heat transfer rate increases with an increase of the Rayleigh number and the solid volume fraction.

Jahanshahi et al. [56] examined the influence of uncertainties due to adopting various formulas for the effective thermal conductivity of Silica–water on heat transfer characteristics for natural convection in a square enclosure. An experimental setup was used to extract the conductivity value of nanofluid. They predicted the effective thermal conductivity of the nanofluid using the model proposed by Hamilton et al. [57] and experimental results. Their findings showed a considerable enhancement in heat transfer by increasing the solid volume fraction at any Rayleigh number using experimental thermal conductivity, whereas a decrease in heat transfer using theoretical thermal conductivity.

Kefayati et al. [58] reported the numerical results of natural convection in enclosures using water/SiO_2 nanofluid. The main aim of their study was to identify the ability of Lattice Boltzmann method (LBM) for solving nanofluid and various geometries. The simulations were performed for different Rayleigh numbers, solid volume fraction and aspect ratio of enclosure. Experimental data from Jahanshahi et al. [56] were used to predict thermal conductivity of SiO_2–water nanofluid. It was concluded that the Nusselt number increases with volume fraction for the whole range of Rayleigh numbers and aspect ratios. Also the effect of nanoparticles on heat transfer augments as the enclosure aspect ratio increases.

Aminossadati and Ghasemi [59] presented a numerical investigation to study the natural convection in an isosceles triangular enclosure with a heat source located at its bottom wall and filled with an Ethylene Glycol–Cu nanofluid. The results indicated that the thermal performance of the enclosure is improved with an increase in the Rayleigh number and solid volume fraction. The results also indicated that the variation of heat transfer rate with respect to the enclosure apex angle, heat source position and dimensions is different at low and high Rayleigh numbers. Moreover, the results obtained from the modified and original Maxwell models indicated that the heat transfer rates obtained based on the modified Maxwell model are generally higher than those obtained based on the original Maxwell model.

Shahi et al. [60] analyzed three-dimensional flow and heat transfer in a single ended tube with non-uniform heat input. They assumed that the sealed end of tube to be adiabatic and also the tube opening to be subjected to copper–water nanofluid. The results indicated that the maximum overall mean Nusselt number is obtained at $\phi = 35^\circ$, while the maximum output mass flow rate is increasing function of the inclination angle. It was also indicated that both the Nusselt number and maximum output mass flow rate are increasing function of solid concentration, but the presence of nanoparticles is more effective at the smaller inclination angles.

Ghasemi and Aminossadati [61] conducted a numerical study to investigate the natural convection cooling of an oscillating heat source embedded on the left wall of an enclosure filled with a water-based nanofluids (Cu, Al_2O_3, TiO_2). They examined the effects of Rayleigh number, solid volume fraction, heat source position, type of nanofluids and oscillation period on the performance of the enclosure. The study showed that the addition of nanoparticles into the pure water improves its thermal conductivity and enhances the heat removal from the heat source. At low Rayleigh number, this effect is more noticeable for copper nanoparticles. It was also showed that Cu and TiO_2 are the most and the least effective nanoparticles, respectively, in the heat removal process.

Elhajjar et al. [62] used laws of thermodynamics for specific heat capacity and thermal expansion coefficient to study numerically the Rayleigh–Bénard convection for three types of nanofluids (water-based Al_2O_3, Cu and CuO). It was observed that for a fixed value of the fluid Rayleigh number, the nanofluid Rayleigh number decreases with the volume fraction of nanoparticles, meaning that the nanoparticles delay the onset of convection.

Sheikhzadeh et al. [63] numerically modeled the buoyancy-driven fluid flow and heat transfer in a square cavity with partially active side walls filled with Cu–water nanofluid. They studied the effects of the position of active portions of the side walls, and volume fraction. The result showed that the average Nusselt number increases with increasing both the Rayleigh number and the volume fraction. Moreover, the maximum average Nusselt number for the high and the low Rayleigh numbers occur for the bottom–middle and the middle–middle locations of the thermally active parts, respectively.

Saleh et al. [64] studied the heat transfer enhancement utilizing water–Cu and water–Al_2O_3 nanofluids in a trapezoidal enclosure. Various inclination angles of the sloping wall, solid volume fractions, and Grashof numbers were considered and the flow and temperature fields as well as the heat transfer was analyzed. They found that the structure of the fluid flow within the enclosure depends upon, Grashof number, inclination angle of sloping wall and nanoparticles concentration and type. In addition, the Cu nanoparticles with high volume fraction combines with an acute sloping wall are most effective in enhancing performance of heat transfer rate.

Mahmoodi [65] numerically investigated the free convection fluid flow and heat transfer of various water base nanofluids. The left and right walls of the cavity are maintained at constant temperature, while its top and bottom walls are insulated. A thin heater is located inside the cavity which its location and length. They investigated the effects of the Rayleigh number, position and location of the heater, the volume fraction of the nanoparticles, and various types of nanofluids on the fluid flow and heat transfer. It was observed that for all cases, the rate of heat transfer increases with increase in Rayleigh number and the volume fraction of the nanoparticles. It was also observed that at low Rayleigh numbers the type of nanofluid does not affect the heat transfer rate while at high Rayleigh numbers maximum rate of heat transfer is obtained by Ag–water nanofluid and its minimum is obtained by TiO_2–water nanofluid.

Lai and Yang [66] performed a numerical study to examine the natural convective thermal efficacy of Al_2O_3/water nanofluid for two circumstances, i.e. at fixed Rayleigh number and at fixed temperature difference across the enclosure, using the lattice Boltzmann method. Results indicated that the average Nusselt number increases with the increase of Rayleigh number and nanoparticle volume concentration. In addition, the relative Nusselt number increases with an increasing average fluid temperature and the increase of temperature difference imposed between the side walls. The deviations between the predicted values of thermal conductivity using various models were found not to be remarkable, while significant under-estimations of nanofluid viscosity were revealed using different models.

3.2. Two phase model

Two-phase model was proposed to get realistic results. Tzou [67,68] analyzed analytically the onset of convection in a horizontal layer uniformly heated from below (the Rayleigh–Bénard problem). They used a model proposed by Buongiorno [11] for convective transport in nanofluid incorporating the effects of Brownian diffusion and thermophoresis. In the study of Tzou [67], the case of stress-free boundaries was analyzed. In his other work [68], the combination of one free and one rigid boundary was investigated, and the case of two rigid boundaries was briefly mentioned. On the basis of his calculations, Tzou [67,68] concluded that for nanofluids the critical Rayleigh number (the value of the Rayleigh number at which instability appears) was lower by one or two orders of magnitude than for regular fluids.

The Cheng-Minkowycz problem was solved by Nield and Kuznetsov [69] for natural convective boundary-layer flow in a porous medium saturated by a nanofluid. They included Brownian and thermophoresis effects and similarity solution was presented. In another study [70], they made an analytical study at the onset of convection in a horizontal layer of a porous medium saturated by a nanofluid by including Brownian and thermophoresis. Again, Nield and Kuznetsov [71] presented a work on a linear stability analysis for the onset of natural convection in a horizontal nanofluid layer. The similar case is studied for natural convective boundary-layer flow of a nanofluid past a vertical plate by Kuznetsov and Nield [72]. Khan and Pop [73] published a paper on boundary-layer flow of a nanofluid past a stretching sheet as a first paper in that field. Their model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. They have taken into account the Pr number, Lewis number, Brownian motion numbers, N_b and thermophoresis number, N_t. They indicated that the reduced Nusselt number is a decreasing function of each dimensionless number and Sherwood number is an increasing function of higher Prandtl number and a decreasing function of lower Pr number for each Le, N_b and N_t numbers. Recently, Pakravan and Yaghubi [74] studied the thermophoresis, Brownian and Dufour effect on natural convective heat transfer of nanofluids simultaneously. They showed that the effects of Dufour strongly decreases with temperature and Nusselt number of natural convection of nanofluids increases with mean temperature of mixture.

Recently, Haddad et al. [75] studied numerically the natural convection heat transfer and fluid flow of CuO–water nanofluid in an open cavity heated from the bottom. A two component non-homogenous equilibrioum model was used for the nanofluid, incorporating the effects of Brownian motion and thermophoresis. Variable thermal conductivity and variable viscosity were taken into account. It was showed that higher heat transfer is formed with the presence of Brownian and thermophoresis effect. It was also found that heat transfer decreases with increasing of nanoparticle volume fraction due to increasing of viscosity of the nanofluid.

4. Experimental investigation

Experimental works on natural convection in nanofluid enclosure are very limited due to difficulties of preparation of nanofluid and measurements of the parameters. Putra et al. [76] made an experimental work on natural convection of Al_2O_3 and CuO–water nanofluid inside a horizontal cylinder heated and cooled from the two ends respectively. They found that the presence of nanaoparticles deteriorates the natural convective heat transfer systematically. They observed a systematic degradation of natural convective heat transfer with increasing particle concentration characterized by decreasing Nusselt number for a given Rayleigh number. The decrease was observed to be more severe for aspect ratio of 1.0 than that of 0.5. They ascribed the possible reasons of deterioration to the effects of particle–fluid slip and sedimentation of nanoparticles.

Wen and Ding [77] conducted an experimental study of natural convective heat transfer of TiO_2 (30–40 nm)/water nanofluid in a vessel which was composed of two horizontal aluminum disks of diameter 240 nm and thickness 10 nm separated by a 10 nm gap. The upper surface of the top aluminum disk was exposed to the open space and cooled by compressed air. They investigated both the transient and steady heat transfer coefficient for different concentrations of nanofluids. They found that nanofluids decrease the natural convective heat transfer coefficient. Furthermore, such deterioration increases with nanoparticle concentrations. They attributed the reason of their observation to the particle–particle interaction and modifications of the dispersion properties.

Nnanna [78] presented an experimental system to investigate the natural convective heat transfer in a differentially heated enclosure, filled with Al_2O_3–water. It was observed for small volume fraction $0.2 \leq \phi \leq 2\%$ the presence of nanoparticles does not impede the free convective heat transfer, rather it augments the rate of heat transfer. However, for large volume fraction $\phi > 2\%$, the convective heat transfer coefficient declines due to reduction in the Rayleigh number caused by increase in kinematic viscosity.

Li and Peterson [79] performed an experimental investigation on the natural convection heat transfer characteristics of Al_2O_3/water nanofluids in a vertical cylinder enclosure heated from below. Compared with the base fluid (distilled water), the heat transfer rate across the enclosure was found increasingly deteriorated with the volume fraction of the nanoparticles in the nanofluid. To further explain the possible causes, other than the relatively enhanced viscosity of the nanofluid, for the deteriorated heat transfer efficacy of using the nanofluid, a flow- visualization experiment of submicron polystyrene–water suspension in rectangular enclosure was performed to infer the detrimental influences due to the Brownian motions and thermophoresis movements of the particles.

Ho et al. [80] reported experimental results which illustrated the natural convective heat transfer of Al_2O_3–water nanofluid in a vertical square enclosure. The thermophysical properties including the particle size, dynamic viscosity, density and thermal conductivity were measured as a function of temperature as well as of the volumetric particle fraction in the nanofluid. The experimental results illustrated systematic heat transfer degradation for the nanofluid containing nanoparticles of $\phi \geq 2\%$ over the entire range of the Rayleigh number considered. However, for the nanofluid containing much lower particle fraction of 0.1%, a heat transfer enhancement of around 18% compared with that of water was found to arise in the largest enclosure at sufficiently high Rayleigh number.

Table 3 shows the summary of published experimental and theoretical investigations of the natural convective heat transfer performance of various nanofluids.

5. Recent Studies

Recently, several studies emerged that analyze natural convection heat transfer enhancement using nanofluids. Mahmoodi [90] proposed a numerical investigation to describe the natural convection fluid flow and heat transfer of Cu-water nanofluid inside L-shaped cavities. The effects of the Rayleigh number, the aspect ratio and the volume fraction were investigated. The results indicated that the average Nusselt number, for all ranges of cavities aspect ratio, increases with increase in the Rayleigh number and the solid volume fraction of the nanofluid. Also, it was indicated that the rate of heat transfer was higher for the L-shaped cavities with lower aspect ratio. Fattahi [91] investigated, numerically, natural convection flows in a cavity subject to

Table 3
Summary of experimental and theoretical studies on natural convection of nanofluids.

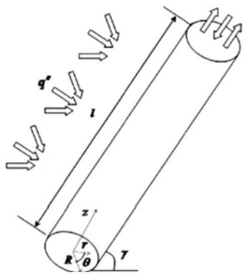
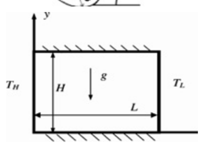
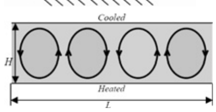
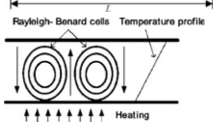
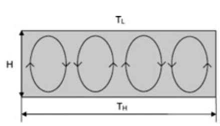
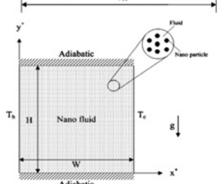
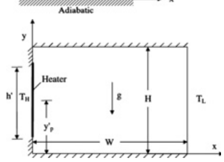
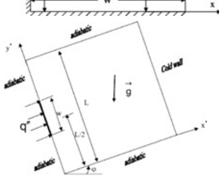
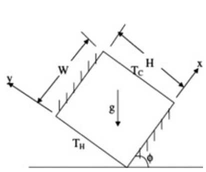
Authors	Configuration	Particle material	Base fluid	Experimental / Numerical method	Parameters	Observations
Shahi et al. [15]		Cu	Water	Finite volume method	$10^3 \leq q_m \leq 10^6$ $0 \leq \phi \leq 5\%$	Both Nusselt number and maximum output mass flow rate are increasing function of solid concentration.
Khanafer et al. [17]		Cu	Water	Finite-volume approach	$10^3 \leq Gr \leq 10^5$ $0 \leq \phi \leq 25\%$ $d_{Cu} = 10$ nm $A = 1.0$	The nanofluid heat transfer rate increases with an increase in the nanoparticles volume fraction. $Nu = 0.5163(0.4436 + \phi^{1.0809})Gr^{0.3123}$
Jang and Choi [21]		Cu/ Diamond	Water	Theoretical study	$300 \leq T \leq 325$ K $d_{Cu} = 6$vnm $d_{diamond} = 2$ nm $\phi \leq 5\%$	The nanofluids are more stable and have larger heat transfer coefficient compared with a classical fluid.
Kim et al. [22]		Cu/Ag	Water	Theoretical study	$0 \leq \phi \leq 20\%$	Heat transfer coefficient of a nanofluid is enhanced by all parameters with respect to the volume fraction.
Jou and Tzeng [24]	Differentially heated cavity	Cu	Water	Finite1difference method	$10^3 \leq Ra \leq 10^6$ $0 \leq \varphi \leq 20\%$ $d_{Cu} = 10$ nm $A = 0.5, 1.0, 2.0$	Increasing the volume fraction of nanofluids cause an increase in the average heat transfer coefficient.
Polidori et al. [25]	Heated vertical plate	γ- Al_2O_3	Water	Theoretical study	$0 \leq \phi \leq 4\%$ $d_{\gamma\text{- Al2O3}} = 42$ nm	The sensitivity to the viscosity model used seems undeniable and plays a key role in the heat transfer behavior.
Hwang et al. [26]		Al_2O_3	Water	Theoretical study	$300 \leq T \leq 325$ K $10 \leq d \leq 50$ nm $\phi \leq 5\%$	The heat transfer is decreased as the size of nanoparticles increases, or the average temperature of nanofluids is decreased.
Ho et al. [29]		Al_2O_3	Water	Finite volume method	$10^3 \leq Ra \leq 10^6$ $0 \leq \phi \leq 4\%$ $A = 1.0$	Significant difference between enhancements in the dynamic viscosity estimated from the two adopted formulas leads to contradictory heat transfer efficacy of the nanofluid $Nu = C(1+\phi)^m Ra^n$
Oztop and Abu-nada [30]		Al_2O_3/ Cu/TiO_2	Water	Finite volume method	$10^3 \leq Ra \leq 5 \times 10^5$ $0 \leq \phi \leq 20\%$ $0.5 \leq A \leq 2$	The heat transfer increases with increasing the value of volume fraction of nanoparticles.
Ögüt [31]		Cu/ Al_2O_3/ CuO/Ag/ TiO_2	Water	PDQ method	$0 \leq \phi \leq 20\%$ $10^4 \leq Ra \leq 10^6$ $0 \leq \varphi \leq 90^\circ$ $A = 1.0$	The average heat transfer rate increases significantly as particle volume fraction increases.
Abu-nada and Oztop [32]		Cu	Water	Finite volume method	$0^\circ \leq \varphi \leq 120^\circ$ $10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 10\%$ $0 \leq \varphi \leq 120^\circ$ $A = 1$	The effect of nanoparticle concentration on Nusselt number is more pronounced at high volume fraction than at low volume fraction.

Table 3 (*continued*)

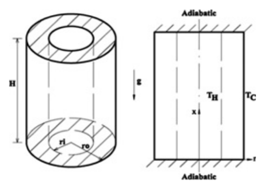
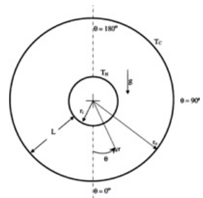
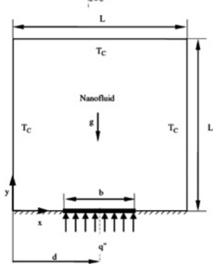
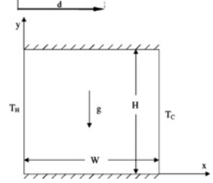
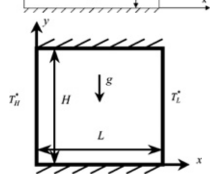
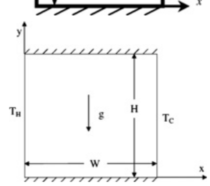
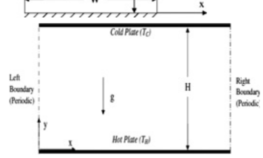
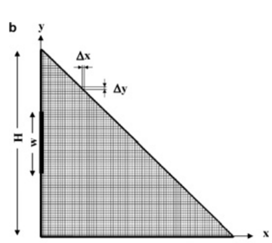
Authors	**Configuration**	**Particle material**	**Base fluid**	**Experimental / Numerical method**	**Parameters**	**Observations**
Abouali and Falahatpisheh [33]		Al_2O_3	Water	Finite volume method	$10^3 \leq Gr \leq 10^5$ $0 \leq \phi \leq 6\%$ $1 \leq A \leq 5$	The thermal conductivity of Jang and Choi [22] agrees well with the experiment and shows deterioration of natural convection heat transfer. $Nu_{nf} = Nu_f(1-c\phi)$
Abu-nada [34]		Al_2O_3	Water	Finite volume method	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 9\%$ $A=0.2, 0.4, 0.8$	For $Ra \geq 10^4$, the average Nusselt number was reduced by increasing the volume fraction of nanoparticles. However, for $Ra=10^3$, the average Nusselt number increased by increasing the volume fraction of nanoparticles.
Aminossadati and Ghasemi [37]		$Cu/TiO_2/Al_2O_3/Ag$	Water	Finite volume approach	$10^3 \leq Ra \leq 10^6$ $0 \leq \phi \leq 20\%$ $A=1.0$	Adding nanoparticles into pure water improves its cooling performance especially at low Rayleigh numbers.
Abu-nada et al. [38]		Al_2O_3/ CuO	Water	Finite volume method	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 9\%$ $1/2 \leq A \leq 2$ $d_{CuO}=29$ nm $d_{Al2O3}=47$ nm	The average Nusselt number was more sensitive to the viscosity models than to the thermal conductivity models.
Lin and violi [39]		Al_2O_3	Water	Weighting function method	$10^3 \leq Gr \leq 10^6$ $5 \leq d_{Al2O3} \leq 250$ nm $0 \leq \phi \leq 5\%$ $A=1.0$	Enhancement and mitigation heat transfer can be attributed to the dominant effect of the Brownian motion caused by heat convection.
Abu-nada and Chamkha [42]		CuO	Water/ EG	Finite volume method	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 6\%$ $1/2 \leq A \leq 2.0$	The effects of the viscosity models are more predominant on the behavior of the average Nusselt number than the influence of the thermal conductivity models.
Abu-nada [43]		CuO	Water	Finite volume method	$10^3 \leq Ra \leq 10^6$ $0 \leq \phi \leq 9\%$ $A=2.0$	For $Ra > 10^3$, the average Nusselt number was reduced by increasing the volume fraction of nanoparticles. However, for $Ra=10^3$, the average Nusselt number was enhanced by increasing the volume fraction of nanoparticles
Corcione [45]	Differentially heated enclosure	$Cu/TiO_2/Al_2O_3$	Water/ EG	Theoretical study	$249 \leq T \leq 324$ K $25 \leq d \leq 150$ nm $1 \leq A \leq 40$ $\phi \leq 6\%$	The optimal volume fraction is found to increase slightly with decreasing the nanoparticle size, and to increase much more remarkably with increasing both the nanofluid average temperature and the slenderness of the enclosure.
Ghasemi and Aminossadati [46]		CuO	Water	Finite volume method	$10^3 \leq Ra \leq 10^6$ $1 \leq \phi \leq 4\%$ $1/2 \leq A \leq 2.0$	When Brownian motion is neglected, the heat transfer rate continuously increases with increasing the solid volume fraction at all Rayleigh numbers.

Table 3 (*continued*)

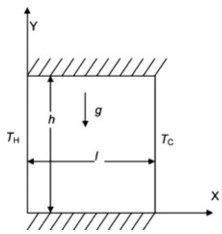
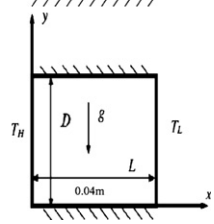
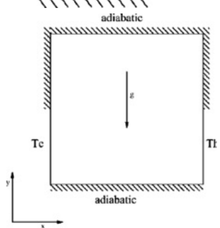
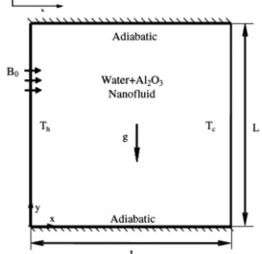
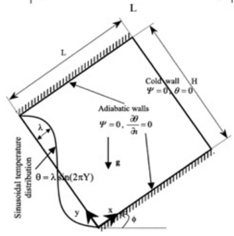
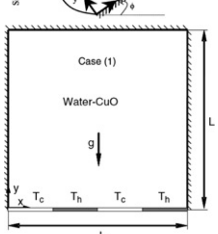
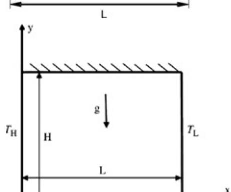
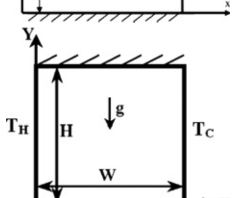
Authors	Configuration	Particle material	Base fluid	Experimental / Numerical method	Parameters	Observations
Santra et al. [49]		Cu	Water	Finite volume method	$10^4 \leq Ra \leq 10$ $0 \leq \phi \leq 5\%$ $d_{Cu} = 100$ nm $A = 1.0$	The heat transfer rate decreases with increase in solid volume fraction for a particular Rayleigh number. However, it increases with Rayleigh number for a particular solid volume fraction.
Rashmi et al. [51]		Al_2O_3	Water	Fluent software	$\phi = 1\%, 4\%$ $A = 1.0$	The rate of heat transfer decreases with increase in particle volume fraction.
Das and Ohal [52]		Cu	Water	Finite volume method	$10^4 \leq Gr \leq 10^7$ $0 \leq \phi \leq 20\%$ $A = 1.0$	The rate of heat transfer increases with an increase in the nanoparticles volume fraction.
Ghasemi et al. [53]		Al_2O_3	Water	Finite volume method	$10^3 \leq Ra \leq 10^7$ $10^3 \leq Ha \leq 10^7$ $0 \leq \phi \leq 6\%$ $A = 1.0$	The heat transfer rate increases with an increase of the Rayleigh number but it decreases with an increase of the Hartmann number.
Oztop et al. [54]		Cu/TiO_2/ Al_2O_3	Water	Finite volume method	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 10\%$ $0 \leq \varphi \leq 90\ ^o$ $A = 1.0$	An enhancement in heat transfer rate was registered for the whole range of Rayleigh numbers.
Aminossadati and Ghasemi [55]		CuO	Water	Finite volume method	$10^3 \leq Gr \leq 10^6$ $0 \leq \phi \leq 4\%$ $A = 1.0$	The heat transfer rate increases with an increase of the Rayleigh number and the solid volume fraction.
Jahanshahi et al. [56]		SiO_2	Water	Finite volume method/ Experimental	$10^5 \leq Ra \leq 10^7$ $0 \leq \phi \leq 4\%$ $A = 1.0$ $d_{SiO2} = 12$ nm	The mean Nusselt number increases with volume fraction for the whole range of Rayleigh numbers. Although by using the theoretical formulations for conductivity no enhancement was observed.
Kefayati et al. [58]		SiO_2	Water	Lattice Boltzmann method	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 4\%$ $1/2 \leq A \leq 2.0$	The Nusselt number increases with volume fraction for the whole range of Rayleigh numbers and aspect ratios.

Table 3 (*continued*)

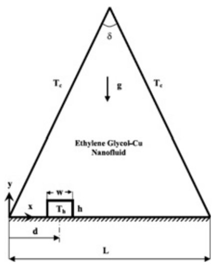
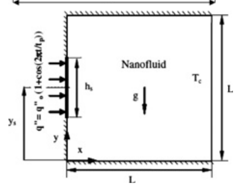
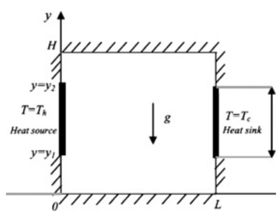
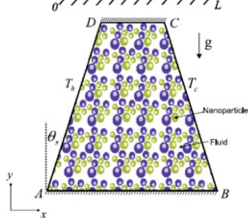
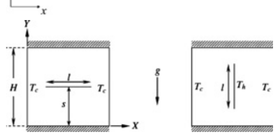
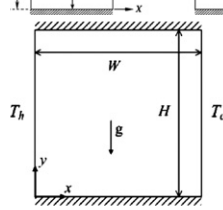
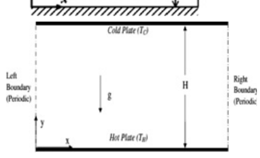
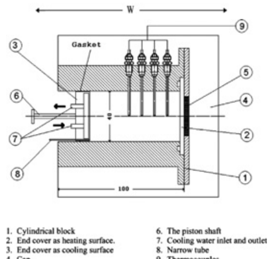
Authors	Configuration	Particle material	Base fluid	Experimental / Numerical method	Parameters	Observations
Aminossadati and Ghasemi [59]		Cu	EG	Finite volume method	$10^3 \le \mathrm{Ra} \le 10^6$ $0 \le \phi \le 5\%$	The thermal performance of the enclosure is improved with an increase in the Rayleigh number and solid volume fraction.
Ghasemi and Aminossadati [61]		$Cu/TiO_2/Al_2O_3$	Water	Finite volume method	$10^3 \le \mathrm{Ra} \le 10^6$ $0 \le \phi \le 20\%$ $A = 1.0$	The addition of nanoparticles into the pure water improves its thermal conductivity and enhances the heat removal from the heat source.
Elhajjar et al. [62]	Rayleigh-Bénard configuration	$Cu/TiO_2/Al_2O_3$	Water		$0 \le \mathrm{Ra} \le 10^4$ $0 \le \phi \le 8\%$	The nanofluid Rayleigh number decreases with the volume fraction of nanoparticles.
Sheikhzadeh et al. [63]		Cu	Water	Finite volume method	$10^3 \le \mathrm{Ra} \le 10^6$ $0 \le \phi \le 15\%$	The average Nusselt number increases with increasing both the Rayleigh number and the volume fraction. $Nu_{avg} = (0.4778\phi^{1.0} + 0.3554)Ra^{0.25}$
Saleh et al. [64]		Cu/Al_2O_3	Water	Finite difference method	$10^3 \le \mathrm{Gr} \le 10^5$ $0 \le \phi \le 26\%$ $0^\circ \le \varphi \le 26^\circ$	The Cu nanoparticles with high volume fraction combines with an acute sloping wall are most effective in enhancing performance of heat transfer rate. $Nu_{avg} = 0.231Gr_{bf}^{(0.342/v)}\left(\frac{k_{nf}\mu_{bf}}{k_{bf}\mu_{nf}}\right)^{-11.707}$
Mahmoodi [65]		$Cu/TiO_2/Ag$	Water	Finite volume method	$10^3 \le \mathrm{Ra} \le 10^6$ $0 \le \phi \le 9\%$ $A = 1.0$	Acute sloping wall and Cu nanoparticles with high concentration are effective to enhance the rate of heat transfer.
Lai and Yang [66]		Al_2O_3	Water	Lattice Boltzmann method	$10^3 \le \mathrm{Ra} \le 10^6$ $0 \le \phi \le 4\%$ $0.006 \le \Delta T \le 10.5\mathrm{k}$ $A = 1.0$	The average Nusselt number increases with the increase of Rayleigh number Ra and nanoparticle volume concentration.
Z. Haddad et al. [75]		CuO	Water	Finite volume method	$10^3 \le \mathrm{Ra} \le 10^6$ $0 \le \phi \le 9\%$ $A = 2.0$	Higher heat transfer is formed with the presence of Brownian and thermophoresis effect.
Putra et al. [76]		CuO/Al_2O_3	Water	Experimental study	$10^7 \le \mathrm{Ra} \le 10^9$ $\phi = 1\%, 4\%$ $d_{Al2O3} = 131.2$ nm $d_{CuO} = 87.3$ nm $A = 0.5, 1.0$	A systematic degradation of natural convective heat transfer with increasing particle concentration for a given Rayleigh number.

Table 3 (*continued*)

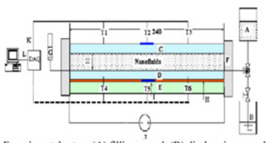
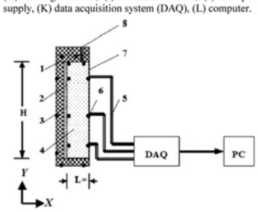
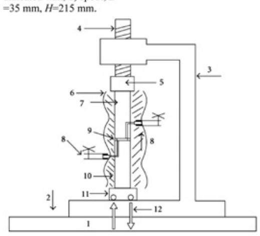
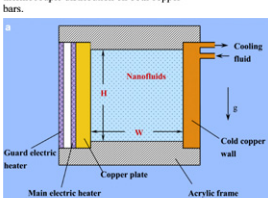
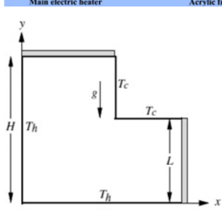
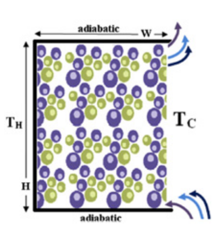
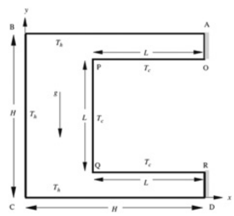
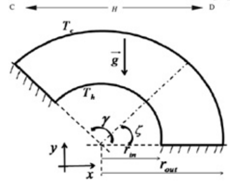
Authors	Configuration	Particle material	Base fluid	Experimental / Numerical method	Parameters	Observations
Wen and Ding [77]	 Experimental setup: (A) filling vessel, (B) discharging vessel, (C) top aluminium plate, (D) bottom aluminium plate, (E) insulation material, (F) insulation cover, (G) level indicator, (H) heating element, (I) heat flux sensor, (J) DC power supply, (K) data acquisition system (DAQ), (L) computer.	TiO_2	Water	Experimental study	$d_{SiO2}=$ 30–40 nm $A=0.5$, 1.0 $0\le\phi\le1\%$ $10^4\le Ra\le10^6$	Nanofluids decrease the natural convective heat transfer coefficient, such deterioration increases with nanoparticle concentrations.
Nnanna [78]	 Experimental setup.1) insulation, 2) resistive electrical heater, 3) heated wall, 4) nanofluid medium, 5) thermocouple wire, 6) the black dots indicate the thermocouple locations, 7) unheated wall, 8) spout; *L* =35 mm, *H*=215 mm.	Al_2O_3	Water	Experimental study	$1\times10^7\le Ra\le3\times10^7$ $0\le\phi\le8\%$ $d_{Al2O3}=27$ nm $A=1.0$	Nusselt increased with small volume fraction,$0.2\le\phi\le2\%$, and no significant change in Nusselt number is observed in the range $2\%\le\phi\le7.9\%$. $Nu=\Gamma e^{-Ra\lambda\phi(e^{-m\phi})}$
Li and Peterson [79]	 Experimental test facility: (Left) 1: horizontal plane table; 2: base plate; 3: vertical support; 4: load screw; 5: heat dissipater; 6: Thermal insulation wrapper; 7: upper copper bar; 8: sample charge tubes; 9: o-ring; 10: lower copper bar; 11: heat sink; 12: coolant inlet and outlet; (Right) the thermocouple distribution on both copper bars.	Al_2O_3	Water	Experimental study	$0.5\%\le\phi\le6\%$ $d_{Al2O3}=47$ nm $8\times10^3\le Ra\le2.8\times10^4$	The heat transfer rate was found increasingly deteriorated with the volume fraction of the nanoparticles.
Ho et al. [80]		Al_2O_3	Water	Experimental study	$6.21\times10^5\le Ra\le2.56\times10^8$ $0.1\%\le\phi\le4\%$	Systematic heat transfer degradation for the nanofluid containing nanoparticles of $\phi\ge2\%$ over the entire range of Rayleigh number.
Mahmoodi [90]		Cu	Water	Finite volume method Simpler algorithm	$10^3\le Ra\le10^6$ $0\le\phi\le10\%$ $A=0.2,0.4,0.6$	The average Nusselt number increases with increase in the solid volume fraction of the nanofluid.
Fattahi [91]	Differentially heated cavity	Cu/Al_2O_3	Water	Lattice Boltzmann method	$10^3\le Ra\le10^6$ $0\le\phi\le5\%$	The average Nusselt number increases for both nanofluids. Further, the effects of solid volume fraction for Cu are stronger than Al_2O_3.
Kefayati et al. [92]		Cu	Water	Lattice Boltzmann method	$10^4\le Ra\le10^6$ $0\le\phi\le5\%$ $1/2\le A\le2.0$	The average Nusselt number increases with augmentation of Rayleigh number and the volume fraction of nanoparticles for whole ranges of aspect ratios.
Abouali and Ahmadi [93]	Six different enclosure shapes	CuO/ Al_2O_3	Water	Fluent software	–	The numerical simulations for natural convection of nanofluids could be obtained theoretically in a much simpler way.
Guo et al. [94]	Differentially heated cavity	Al_2O_3	Water	Lattice Boltzmann method	$10^3\le Ra\le10^5$ $0\le\phi\le15\%$ $A=1.0$	The critical Rayleigh number is lower for nanofluid than that for base fluid. $Nu=0.05Ra^{0.45}$

Table 3 (*continued*)

Authors	Configuration	Particle material	Base fluid	Experimental / Numerical method	Parameters	Observations
Mahmoodi and Hashemi [95]		Cu	Water	Finite volume method Simpler algorithm	$10^3 \leq Ra \leq 10^6$ $0 \leq \phi \leq 10\%$ $A=0.2,0.4,0.6,0.8$	The mean Nusselt number increased with increase in volume fraction regardless aspect ratio of the enclosure.
Soleimani et al. [96]		Cu	Water	Finite Element Method	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 6\%$ $45^\circ \leq \varphi \leq 180^\circ$ $A=1.0$	The effect of nanoparticles is more pronounced at low Rayleigh number than at high Rayleigh number.

different side wall temperatures. The fluid in the cavity was a water-based nanofluid containing Al_2O_3 and Cu nanoparticles. The effective thermal conductivity and viscosity of nanofluid were calculated using Chon and Brinkman models. The results indicated that by increasing solid volume fraction, the average Nusselt number increased for both nanofluids. Kefayati et al. [92] numerically studied natural convection of Cu-water nanofluid in an open enclosure using Lattice Boltzmann method (LBM). The results indicated that the average Nusselt number increased with augmentation of the Rayleigh number and the volume fraction of nanoparticles for the whole range of aspect ratios. However, the average Nusselt number decreased as the aspect ratio increased. Abouali and Ahmadi [93] analyzed natural convection of nanofluids in horizontal and titled square, horizontal and vertical annuli, a triangular enclosure and the Rayleigh-Bénard convection configuration. The study was performed for Al_2O_3 and CuO nanofluids. The results were compared with the prediction of a theoretical approach which uses the Nusselt correlations available in literature for pure fluids. Guo et al. [94] conducted a numerical study for natural convection heat transfer in a square cavity using the LBM. They derived a correlation for the mean Nusselt number. Mahmoodi and Hashemi [95] presented a numerical study of natural convection fluid flow and heat transfer inside C-shaped enclosures filled with Cu-water nanofluid using finite volume method. It was found that the mean Nusselt number increases with increase in the Rayleigh number and volume fraction of Cu nanoparticles regardless of aspect ratio of the enclosure. Also, the effect of nanoparticles on enhancement of heat transfer at low Rayleigh numbers is more significant than that at high Rayleigh numbers. Moreover, it was shown that the rate of heat transfer increases as the C-shaped enclosure becomes narrower. Soleimani et al. [96] proposed the control volume based finite element method (CVFEM) to investigate natural convection heat transfer in a semi-annulus enclosure filled with Cu-water nanofluid. The inner and outer semi circular walls are maintained at constant temperatures while the two other walls are thermally insulated. The numerical investigation is carried out for different governing parameters: namely the Rayleigh number, nanoparticle volume fraction and the angle of turn for the enclosure. The effective thermal conductivity and viscosity of nanofluid were calculated using the Maxwell-Garnetts (MG) and Brinkman models, respectively. It was noticed that the streamlines and isotherms strongly depend on the volume fraction of nanoparticle.

6. Conclusion

This paper presented a comprehensive review for the recent work published on heat transfer enhancement in natural convection using nanofluids. The paper presented experimental, numerical and analytical published work in literature. The numerical work included single-phase and two-phase models. Various thermal conductivity and viscosity models employed by various researchers were presented.

The literature survey shows that natural convection heat transfer enhancement using nanofluid is still controversial and there is an on-going debate on the role of nanoparticles on the heat transfer enhancement since there is a lack of extensive studies on this topic. Clearly, most of numerical results showed that nanofluids significantly improve the heat transfer capability of conventional heat transfer fluid. Whereas, experimental results showed that presence of nanoparticles deteriorates heat transfer systematically. It was observed that the experimental results were mostly obtained using one type of nanofluid (Al_2O_3–water). Thus, benchmark experiments are highly desired to gage the validity of the numerical results. Concerning the numerical studies which try to explain the observed anomalous enhancement in heat transfer, it will be necessary to consider not only one single phase model but two-phase approach, which seems a better model to describe the nanofluid flow since slip velocity between the particle and base fluid plays important role on the heat transfer performance of nanofluids.

References

[1] Eastman JA, Choi SUS, Li S, Yu W, Thompson LJ. Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. Applied Physics Letters 2001;78:718–20.
[2] Choi US. Enhancing thermal conductivity of fluids with nanoparticles. ASME Fluids Engineering Division 1995;231:99–105.
[3] Keblinski P, Phillpot SR, Choi SUS, Eastman JA. Mechanisms of heat flow in suspensions of nano-sized particles (nanofluids). International Journal of Heat and Mass Transfer 2002;45:855–63.
[4] Wang XW, Xu XF, Choi SUS. Thermal conductivity of nanoparticle–fluid mixture. Journal of Thermophysics and Heat Transfer 1999;13:474–80.
[5] Xie HQ, Wang JC, Xi TG, Li Y, Ai F. Dependence of the thermal conductivity of nanoparticle–fluid mixture on the base fluid. Journal of Materials Science Letters 2002;21:1469–71.
[6] Xuan Y, Li Q. Heat transfer enhancement of nanofluids. International Journal of Heat and Fluid Flow 2000;21:58–64.

[7] Xuan Y, Roetzel W. Conceptions for heat transfer correlations of nanofluids. International Journal of Heat and Mass Transfer 2000;43:3701–7.
[8] Bianco V, Manca O, Nardini S. Numerical investigation on nanofluids turbulent convection heat transfer inside a circular tube. International Journal of Thermal Sciences 2009;29:3632–42.
[9] Mirmasoumi S, Behzadmehr A. Numerical study of laminar mixed convection of a nanofluid in a horizontal tube using two-phase mixture model. Applied Thermal Engineering 2008;28:717–27.
[10] Akbarinia A, Laur R. Investigation the diameter of solid particles affects on a laminar nanofluid flow in a curved tube using a two phase approach. International Journal of Heat and Fluid Flow 2009;30:706–14.
[11] Buongiorno J. Convective transport in nanofluids. ASME. Journal of Heat Transfer 2006;128:240–50.
[12] Pak BC, Cho YI. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. Experimental Heat Transfer 1998;11: 151–70.
[13] Maiga SEB, Nguyen CT, Galanis N, Roy G. Heat transfer behaviours of nanofluids in a uniformly heated tube. Superlattices Microstructures 2004;35:543–57.
[14] Palm SJ, Roy G, Nguyen CT. Heat transfer enhancement with the use of nanofluids in radial flow cooling systems considering temperature dependent properties. Applied Thermal Engineering 2006;26:2209–18.
[15] Das SK, Choi SUS, Yu W, Pradeep T. Nanofluids Science and Technology. Hoboken, NJ: John Wiley & Sons; 2008.
[16] Eastman JA, Choi SUS, Li S, Soyez G, Thompson LJ, Di Melfi RJ. Novel thermal properties of nanostructured materials. Material Science Forum 1999;312–314:629–34.
[17] Khanafer K, Vafai K, Lightstone M. Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. International Journal of Heat and Mass Transfer 2003;46:3639–53.
[18] Wasp FJ. Solid–liquid slurry pipeline transportation. Berlin: Trans Tech; 1977.
[19] Brinkman HC. The viscosity of concentrated suspensions and solutions. The Journal of Chemical Physics 1952;20:571–81.
[20] Jang SP, Choi SUS. Free convection in a rectangular cavity (Benard convection) with nanofluids, Proceedings of IMECE04. Anaheim, California, USA; 2004. pp. 1–7.
[21] Jang SP, Choi SUS. The role of Brownian motion in the enhanced thermal conductivity of nanofluids. Applied Physics Letters 2004;84:4316–8.
[22] Kim J, Kang YT, Choi CK. Analysis of convective instability and heat transfer characteristics of nanofluids. Physics of Fluids 2004;16:2395–401.
[23] Wang BX, Zhou LR, Peng XF. A fractal model for predicting the effective thermal conductivity of liquid with suspension of nanoparticles. International Journal of Heat and Mass Transfer 2003;46:2665–72.
[24] Jou RY, Tzeng SC. Numerical research of nature convective heat transfer enhancement filled with nanofluids in rectangular enclosures. International Communications in Heat and Mass Transfer 2006;33:727–36.
[25] Polidori G, Fohanno S, Nguyen CT. A note on heat transfer modeling of Newtonian nanofluids in laminar free convection. International Journal of Thermal Sciences 2007;46:739–44.
[26] Hwang KS, Lee JH, Jang JP. Buoyancy-driven heat transfer of water-based Al_2O_3 nanofluids in a rectangular cavity. International Journal of Heat and Mass Transfer 2007;50:4003–10.
[27] Einstein A. Investigation on the Theory of Brownian Motion. New York: Dover; 1956.
[28] Pak BC, Cho Y. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particle. Experimental Heat Transfer 1998;11:151–70.
[29] Ho CJ, Chen MW, Li ZW. Numerical simulation of natural convection of nanofluid in a square enclosure: effects due to uncertainties of viscosity and thermal conductivity. International Journal of Heat and Mass Transfer 2008;51:4506–16.
[30] Oztop HF, Abu-Nada E. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. International Journal of Heat and Fluid Flow 2008;29:1326–36.
[31] Ögüt EB. Natural convection of water-based nanofluids in an inclined enclosure with a heat source. International Journal of Thermal Sciences 2009;48:2063–73.
[32] Abu-Nada E, Oztop HF. Effects of inclination angle on natural convection in enclosures filled with Cu–water nanofluid. International Journal of Heat and Fluid Flow 2009;30:669–78.
[33] Abouali O, Falahatpisheh A. Numerical investigation of natural convection of Al_2O_3 nanofluids in vertical annuli. Journal of Heat and Mass Transfer 2009;46:15–23.
[34] Abu-Nada E. Effects of variable viscosity and thermal conductivity of Al_2O_3-water nanofluid on heat transfer enhancement in natural convection. International Journal of Heat and Fluid Flow 2009;30:679–90.
[35] Chon CH, Kihm KD, Lee SP, Choi SUS. Empirical correlation finding the role of temperature and particle size for nanofluid (Al_2O_3) thermal conductivity enhancement. Applied Physics Letters 2005:87 Article ID 153107.
[36] Nguyen CT, Desgranges F, Roy G, Galanis N, Mare T, Boucher S, Angue Minsta H. Temperature and particle-size dependent viscosity data for water-based nanofluids–hysteresis phenomenon. International Journal of Heat and Fluid Flow 2007;28:1492–506.
[37] Aminossadati SM, Ghasemi B. Natural convection cooling of a localised heat source at the bottom of a nanofluid-filled enclosure. European Journal of Mechanics B/Fluids 2009;28:630–40.
[38] Abu-Nada E, Masoud Z, Oztop HF, Compo A. Effect of nanofluid variable properties on natural convection in enclosures. International Journal of Thermal Science 2010;49:479–91.
[39] Lin KC, Violi A. Natural convection heat transfer of nanofluids in a vertical cavity: effects of non-uniform particle diameter and temperature on thermal conductivity. International Journal of Heat and Fluid Flow 2010;31:236–45.
[40] Xu J, Yu B, Zou M, Xu P. A new model for heat conduction of nanofluids based on fractal distributions of nanoparticles. Journal of Physics D 2006;39:4486–90.
[41] Jang SP, Lee JH, Hwang KS, Choi SUS. Particle concentration and tube size dependence of viscosities of Al_2O_3-water nanofluids flowing through micro- and minitubes. Applied Physics Letters 2007;91:243112.
[42] Abu-Nada E, Chamkha A. Effect of nanofluid variable properties on natural convection in enclosures filled with a CuO–EG–water nanofluid. International Journal of Thermal Science 2010;49:2339–52.
[43] Abu-Nada E. Rayleigh-Bénard convection in nanofluids: Effect of temperature dependent properties. International Journal of Thermal Science 2011;50:1720–30.
[44] Corcione M. Empirical correlating equations for predicting the effective thermal conductivity and dynamic viscosity of nanofluids. Energy Conversion and Management 2011;52:789–93.
[45] Corcione M. Heat transfer features of buoyancy-driven nanofluids inside rectangular enclosures differentially heated at the sidewalls. International Journal of Thermal Science 2010;49:1536–46.
[46] Ghasemi B, Aminossadati SM. Brownian motion of nanoparticles in a triangular enclosure with natural convection. International Journal of Thermal Science 2010;49:931–40.
[47] Koo J, Kleinstreuer C. Laminar nanofluid in microheat-sinks. International Journal of Heat and Mass Transfer 2005;48:2652–61.
[48] Koo J, Kleinstreuer C. A new thermal conductivity model for nanofluids. Journal of Nanoparticle Research 2004;6:577–88.
[49] Santra AK, Sen S, Chakraborty N. Study of heat transfer augmentation in a differentially heated square cavity using copper-water nanofluid. International Journal of Thermal Science 2008;47:1113–22.
[50] Bird RB, Stewert WE, Lightfoot EN. Transport Phenomena. Singapore: John Wiley & Sons; 1960.
[51] Rashmi W, Ismail AF, Khalid M. CFD studies on natural convection heat transfer of Al_2O_3–water nanofluids. Heat and Mass Transfer, (accepted April 2010) http://dx.doi.org/10.1007/s00231-011-0792-x.
[52] Das MK, Ohal PS. Natural convection heat transfer augmentation in a partially heated and partially cooled square cavity utilizing nanofluids. International Journal of Numerical Methods for Heat and Fluid Flow 2009;19:411–31.
[53] Ghasemi B, Aminossadati SM, Raisi A. Magnetic field effect on natural convection in a nanofluid-filled square enclosure. International Journal of Thermal Science 2011;50:1748–56.
[54] Oztop HF, Abu-Nada E, Varol Y, Al-Salem K. Computational analysis of non-isothermal temperature distribution on natural convection in nanofluid filled enclosures. Superlattices Microstructures 2011;49:453–67.
[55] Aminossadati SM, Ghasemi B. Natural convection of water–CuO nanofluid in a cavity with two pairs of heat source–sink. International Communications in Heat and Mass Transfer 2011;38:672–8.
[56] Jahanshahi M, Hosseinizadeh SF, Alipanah M, Dehghani A, Vakilinejad GR. Numerical, Simulation of free convection based on experimental measured conductivity in a square cavity using water/SiO_2 nanofluid. International Communications in Heat and Mass Transfer 2010;37:687–94.
[57] Hamilton RL, Crosser OK. Thermal conductivity of heterogeneous two component systems. I&EC Fundamentals 1962;1:187–91.
[58] Kefayati 0, Hosseinizadeh SF, Gorji M, Sajjadi H. Lattice Boltzmann simulation of natural convection in tall enclosures using water/SiO_2 nanofluid. International Communications in Heat and Mass Transfer 2011;38:798–805.
[59] Aminossadati SM, Ghasemi B. Enhanced natural convection in an isosceles triangular enclosure filled with a nanofluid. Computers Mathematics Applications 2011;61:1739–53.
[60] Shahi M, Mahmoudi AH, Talebi F. Numerical simulation of steady natural convection heat transfer in a 3-dimensional single-ended tube subjected to a nanofluid. International Communications in Heat and Mass Transfer 2010;37: 1535–45.
[61] Ghasemi B, Aminossadati SM. Periodic natural convection in a nanofluid-filled enclosure with oscillating heat flux. International Journal of Thermal Science 2010;49:1–9.
[62] Elhajjar B, Bachira G, Mojtabi A, Fakihc C, Charrier-Mojtabi MC. Modeling of Rayleigh–Bénard natural convection heat transfer in nanofluids. C R Mecanique 2010;338:350–4.
[63] Sheikhzadeh GA, Arefmanesh A, Kheirkhah MH, Abdollahi R. Natural convection of Cu–water nanofluid in a cavity with partially active side walls. European Journal of Mechanics B/Fluids 2011;30:166–76.
[64] Saleh H, Roslan R, Hashim I. Natural convection heat transfer in a nanofluid-filled trapezoidal enclosure. International Journal of Heat and Mass Transfer 2011;54:194–201.
[65] Mahmoodi M. Numerical simulation of free convection of nanofluid in a square cavity with an inside heater. International Journal of Thermal Sciences 2011:1–15.
[66] Lai FH, Yang YT. Lattice Boltzmann simulation of natural convection heat transfer of Al_2O_3/water nanofluids in a square enclosure. International Journal of Thermal Sciences 2011;50:1930–41.
[67] Tzou DY. Instability of nanofluids in natural convection. ASME Journal of Heat Transfer 2008;130:072401.

[68] Tzou DY. Thermal instability of nanofluids in natural convection. International Journal of Heat and Mass Transfer 2008;51:2967–79.
[69] Nield DA, Kuznetsov AV. The Cheng-Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid. International Journal of Heat and Mass Transfer 2009;52:5792–5.
[70] Nield DA, Kuznetsov AV. The onset of convection in a horizontal nanofluid layer of finite depth. Europian Fluid Mechanics B/Fluids 2010;29: 217–23.
[71] Nield DA, Kuznetsov AV. Thermal instability in a porous medium layer saturated by a nanofluid. International Journal of Heat and Mass Transfer 2009;52:5796–801.
[72] Kuznetsov AV, Nield DA. Natural convective boundary-layer flow of a nanofluid past a vertical plate. International Journal of Thermal Sciences 2010;49:243–7.
[73] Khan WA, Pop I. Boundary-layer flow of a nanofluid past a stretching sheet. International Journal of Heat and Mass Transfer 2010;53:2477–83.
[74] Pakravan HA, Yaghoubi M. Combined thermophoresis, Brownian motion and Dufour effects on natural convection of nanofluids. International Journal of Thermal Sciences 2011;50:394–402.
[75] Haddad Z, Oztop HF, Abu-Nada E, Mataoui A. Natural Convection in Nanofluids: Are the thermophoresis and Brownian motion effects significant in nanofluid heat transfer enhancement? International Journal of Thermal Sciences 2012;57:152–62.
[76] Putra N, Roetzel W, Das SK. Natural convection of nanofluids. Heat and Mass Transfer 2003;39:775–84.
[77] Wen D, Ding Y. Formulation of nanofluids for natural convective heat transfer applications. International Journal of Heat and Fluid Flow 2005;26: 855–64.
[78] Nnanna AGA. Experimental model of temperature-driven nanofluid. Journal of Heat Transfer-Transactions of the ASME 2007;129:697–704.
[79] Li CH, Peterson GP. Experimental studies of natural convection heat transfer of Al_2O_3/DI water nanoparticle suspensions (nanofluids). Advances in Mechanical Engineering 2010 Article ID742739.
[80] Ho CJ, Liu WK, Chang YS, Lin CC. Natural convection heat transfer of alumina-water nanofluid in vertical square enclosures: an experimental study. International Journal of Thermal Sciences 2010;49:1345–53.
[81] Maïga SEB, Palm SM, Nguyen CT, Roy G, Galanis N. Heat transfer enhancement by using nanofluids in forced convection flows. International Journal of Heat and Fluid Flow 2005;26:530–46.
[82] Orozco D. Hydrodynamic behavior of suspension of polar particles. Encyclopedia Surface Colloid Science 2005;4:2375–96.
[83] Nguyen CT, Desgranges F, Galanis N, Roy G, Mare T, Boucher S, Mintsa HA. Viscosity data for Al_2O_3-water nanofluid-hysteresis: is heat transfer enhancement using nanofluids reliable? International Journal of Thermal Science 2008;47:103–11.
[84] Gherasim I, Roy G, Nguyen CT, Vo-Ngoc D. Experimental investigation of nanofluids in confined laminar radial flows. International Journal of Thermal Science 2009;48:1486–93.
[85] Charuyakorn P, Sengupta S, Roy SK. Forced convection heat transfer in micro encapsulated phase change material slurries. International Journal of Heat and Mass Transfer 1991;34:819–33.
[86] Eastman JA, Phillpot SR, Choi US, Keblinski P. Thermal transport in nanofluids. Annual Review of Materials Research 2000;34:219–46.
[87] Yu W, Choi SUS. The role of interfacial layers in the enhanced thermal conductivity of nanofluids: a renovated Maxwell model. Journal of Nanoparticle Research 2003;5:167–71.
[88] Patel HE, Sundarrajan T, Pradeep T, Dasgupta A, Dasgupta N, Das SK. A micro-convection model for thermal conductivity of nanofluid. Pramana Journal of Physics 2005;65:863–9.
[89] Angue Mintsa H, Roy G, Nguyen CT, Doucet D. New temperature dependent thermal conductivity data for water-based nanofluids. International Journal of Thermal Science 2009;48:363–71.
[90] Mahmoodi M. Numerical simulation of free convection of a nanofluid in L-shaped cavities. International Journal of Thermal Science 2011;50:1731–40.
[91] Fattahi E, Farhadi M, Sedighi K, Nemati H. Lattice Boltzmann simulation of natural convection heat transfer in nanofluids. International Journal of Thermal Science 2012;52:137–44.
[92] Kefayati GHR, Hosseinizadeh SF, Gorji M, Sajjadi H. Lattice Boltzmann simulation of natural convection in an open enclosure subjugated to water/copper nanofluid. International Journal of Thermal Science 2012;52:91–101.
[93] Abouali O, Ahmadi G. Computer simulations of natural convection of single phase nanofluids in simple enclosure: a critical review. Applied Thermal Engineering 2012;36:1–13.
[94] Guo Y, Qin D, Shen S, Bennacer R. Nanofluid multi-phase convective heat transfer in closed domain: simulation with lattice Boltzmann method. International Communications in Heat and MassTransfer 2012;39:350–4.
[95] Mahmoodi M, Hashemi SM. Numerical study of natural convection of a nanofluid in C-shaped enclosures. International Journal of Thermal Science 2012;55:76–89.
[96] Soleimani S, Sheikholeslami M, Ganji DD, Gorji-Bandpay. Natural convection heat transfer in a nanofluid filled semi-annulus enclosure. International Communications in Heat and Mass Transfer 2012;39:565–74.